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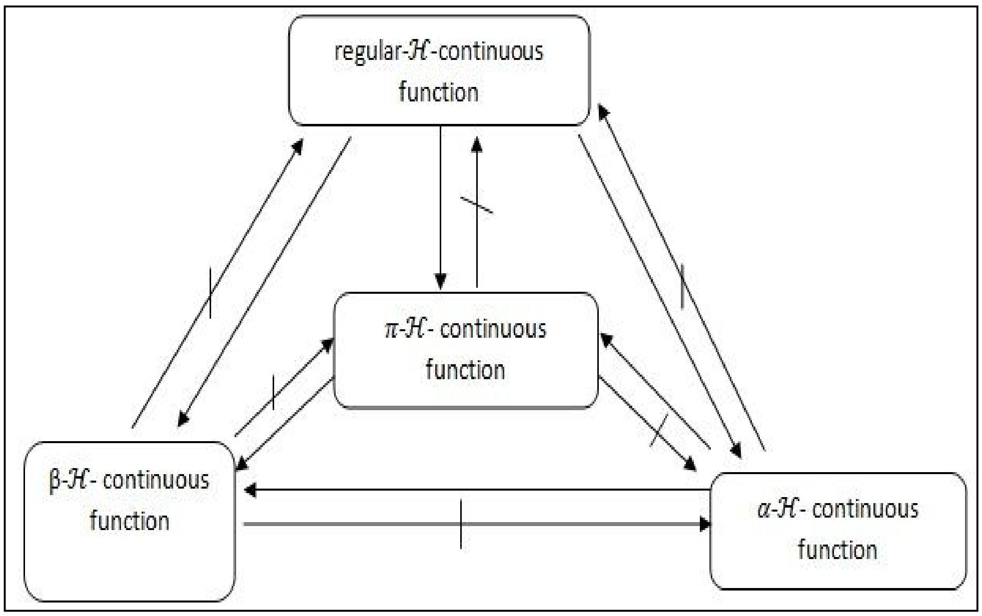
A STUDY ON REGULAR-$\mathcal{H}$-OPEN SETS AND REGULAR-$\mathcal{H}$-CONTINUOUS FUNCTIONS



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ABSTRACT

In this paper the concept of Regular-$\mathcal{H}$-open sets is introduced and its inter relations with other types of $\mathcal{H}$-open sets are studied with suitable counter examples. Equivalently the inter relations of Regular-$\mathcal{H}$-continuous functions with other types of Hereditary continuous functions are discussed with necessary counter examples.

KEY WORDS : *Regular-$\mathcal{H}$-open sets, $R_{\mathcal{H}}$-open, $R_{\mathcal{H}}$-closed, $R_{\mathcal{H}}$-irresolute functions, Regular-$\mathcal{H}$-continuous functions.*

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1. INTRODUCTION AND PRELIMINARIES

1.1 INTRODUCTION

In 2007, Csaszar [9] defined a nonempty class of subsets of a nonempty set called hereditary class and studied modification of generalized topology via hereditary classes. Csaszar [9] introduced the notions of hereditary generalized topological space. The aim of this paper is to extend the study of the properties of the generalized topologies via hereditary classes. In this paper the concept of Regular-$\mathcal{H}$-open sets is introduced and its inter relations with other types of $\mathcal{H}$-open sets are studied with suitable counter examples. Equivalently the inter relations of Regular-$\mathcal{H}$-continuous functions with other types of Hereditary continuous functions are discussed with necessary counter examples.

1.2 PRELIMINARIES

Definition 1.2.1 A subset A of a topological space (X, μ) is called **pre open set [21]** if A $\subseteq$ int(cl(A)), **α-open set [20]** if A $\subseteq$ int(cl(int(A))), **β-open set [1]** if A $\subseteq$ cl(int(cl(A))), **regular open set [22]** if A = int(cl(A)), ***π*-open set [23]** if the finite union of regular open sets.

Definition 1.2.2 [9] Let X be a nonempty set and let expX be the power of X. The collection μ of subset of X satisfying the following conditions is called generalized topology $\emptyset \in \mu$, $G_i \in \mu$ for i $\in$ *I* implies G = $\bigcup_{i\in I} G_i \in \mu$.

The elements of μ are called μ-open and their compliments are called μ-closed. The pair (X,μ) is called a generalized topological spaces (GTS).

Definition 1.2.3 [9] Let X be a nonempty set. A hereditary class $\mathcal{H}$ of X if A $\in \mathcal{H}$ and B $\subset$ A then B $\in \mathcal{H}$. A generalized topological spaces (X, μ) with a hereditary class $\mathcal{H}$ is Hereditary Generalized Topological Spaces (HGTS) and denoted by (X, μ, $\mathcal{H}$). For each A $\subset$ X, $A^*(\mathcal{H}, \mu)$ = { x $\in$ X : A $\cap$ G $\notin \mathcal{H}$ for every $G_i \in \mu$ such that x $\in$ G }. If there is no ambiguity then we write A* in place of $A^*(\mathcal{H}, \mu)$. For each A $\subset$ X, then $c^*_\mu(A)$ = A $\cup$ A^*.

Definition 1.2.4 [8] Let X and Y be topological spaces. A function f : X $\rightarrow$ Y is called a open map is f(F) is open in Y whenever F is open in X.

Definition 1.2.5 [8] Let X and Y be topological spaces. A function f : X $\rightarrow$ Y is called a closed map is f(F) is closed in Y whenever F is closed in X.

2. A STUDY ON REGULAR-$\mathcal{H}$-OPEN SETS

Definition 2.1 Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space. Any subset A of X is said to be regular-$\mathcal{H}$-open if A = $i_\mu(c^*_\mu(A))$. The complement of a regular-$\mathcal{H}$-open set is said to be a regular-$\mathcal{H}$-closed.

Definition 2.2 Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space and A be any subset of X. Then the regular $\mathcal{H}$-interior of A (briefly, $R_{\mathcal{H}}$int(A)) is defined by

$$R_{\mathcal{H}}\text{int}(A) = \cup \{ G ; G \subseteq A \text{ and each } G \subseteq X \text{ is a regular } \mathcal{H}\text{-open set} \}.$$

Definition 2.3 Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space and A be any subset of X. Then the regular $\mathcal{H}$-closure of A (briefly, $R_{\mathcal{H}}$cl(A)) is defined by

$$R_{\mathcal{H}}\text{cl}(A) = \cap \{ K ; A \subseteq K \text{ and each } K \subseteq X \text{ is a regular } \mathcal{H}\text{-closed set} \}.$$

Definition 2.4 [10] Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space. Any subset A of X is said to be β-$\mathcal{H}$-open if $A \subseteq c_\mu(i_\mu(c_\mu^*(A)))$. The complement of a β-$\mathcal{H}$-open set is said to be a β-$\mathcal{H}$-closed.

Definition 2.5 [10] Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space. Any subset A of X is said to be π-$\mathcal{H}$ open if $A \subseteq i_\mu(c_\mu^*(A))$. The complement of a π-$\mathcal{H}$-open set is said to be a π-$\mathcal{H}$-closed.

Definition 2.6 [10] Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space. Any subset A of X is said to be α-$\mathcal{H}$-open if $A \subseteq i_\mu(c_\mu^*(i_\mu(A)))$. The complement of an α-$\mathcal{H}$-open set is said to be an α-$\mathcal{H}$-closed.

Note 2.1 Clearly every regular $\mathcal{H}$-open set is μ-open.

Proposition 2.1 Every π-$\mathcal{H}$-open set is β-$\mathcal{H}$-open.

Remark 2.1 The converse of Proposition 2.1 need not be true as shown in Example 2.7.

Example 2.7 Let X = { a, b, c, d, e, f }, μ = { ϕ, { d, e }, { e }, { a, c, e }, { a, c }, { a, c, d, e } } and $\mathcal{H}$ = { ϕ, { a } }. Clearly, μ is a generalized topology and $\mathcal{H}$ is a hereditary class and the triple (X, μ, $\mathcal{H}$) is a hereditary generalized topological space. The μ-closed sets are X, { a, b, c, f }, { a, b, c, d, f }, { b, d, f }, { b, d, e, f } and { b, f }. Let A = { a, c, f } be a subset of X. Then $A^* =$ { a, c, d, e } and so $c_\mu^*(A) = A \cup A^* =$ { a, c, d, e, f }, $i_\mu(c_\mu^*(A)) =$ { a, c, d, e }. Also, $c_\mu(i_\mu(c_\mu^*(A))) = X$. Hence, $\mathbf{A \subset c_\mu(i_\mu(c_\mu^*(A)))}$. Therefore, **A is β- $\mathcal{H}$-open**. But $i_\mu(c_\mu^*(A)) =$ {a, c, d, e}, $\mathbf{A \nsubseteq i_\mu(c_\mu^*(A))}$. Hence **A is not π- $\mathcal{H}$-open**. Therefore A is β-$\mathcal{H}$-open but not π-$\mathcal{H}$-open. Hence every β-$\mathcal{H}$-open set need not be a π-$\mathcal{H}$-open.

Proposition 2.2 Every α-$\mathcal{H}$-open set is π-$\mathcal{H}$-open.

Remark 2.2 The converse of Proposition 2.2 need not be true as shown in Example 2.18.

Example 2.8 Let X = { a, b, c, d, e }, μ = { ϕ, { a }, { c }, { a, c }, { a, b, c }, { c, d }, { a, c, d }, { a, b, c, d } } and $\mathcal{H}$ = { ϕ, { a }, { b }, { c } }. Clearly, μ is a generalized topology and $\mathcal{H}$ is a hereditary class and the triple (X, μ, $\mathcal{H}$) is a hereditary generalized topological space. Let A = { a, b, d } be a subset of X. Then, $A^* =$ { a, b, c, d } and so $c_\mu^*(A) = A \cup A^* =$ { a, b, c, d }. Then, $i_\mu(c_\mu^*(A)) =$ { a, b, c, d }. Hence, $\mathbf{A \subset i_\mu(c_\mu^*(A))}$. Therefore, **A is π-$\mathcal{H}$-open**. But $i_\mu(c_\mu^*(i_\mu(A))) =$ { a }, $\mathbf{A \nsubseteq i_\mu(c_\mu^*(i_\mu(A)))}$. Hence **A is not α-$\mathcal{H}$-open**. Therefore, A is π-$\mathcal{H}$-open but not α-$\mathcal{H}$-open. Hence every π-$\mathcal{H}$-open set need not be a α -$\mathcal{H}$-open.

Proposition 2.3 Every α-$\mathcal{H}$-open set is β-$\mathcal{H}$-open.

Remark 2.3 The converse of Proposition 2.3 need not be true as shown in Example 2.9.

Example 2.9 Let X = { a, b, c, d, e, f }, μ = { ϕ, { d, e }, { e }, { a, c, e }, { a, c }, { a, c, d, e } } and $\mathcal{H}$ = { ϕ, { a } }. Clearly, μ is a generalized topology and $\mathcal{H}$ is a hereditary class and the triple (X, μ, $\mathcal{H}$) is a hereditary generalized topological space. The μ-closed sets are X, { a, b, c, f }, { a, b, c, d, f }, { b, d, f }, { b, d, e, f } and { b, f }. Let A = { a, c, f } be a subset of X. Then, $A^* =$ { a, c, d, e } and so $c_\mu^*(A) = A \cup A^* =$ { a, c, d, e, f }, $i_\mu(c_\mu^*(A)) =$ { a, c, d, e }. Hence, $c_\mu(i_\mu(c_\mu^*(A))) = X$. Hence $\mathbf{A \subset c_\mu(i_\mu(c_\mu^*(A)))}$. Therefore, **A is β- $\mathcal{H}$-open**. But $i_\mu(c_\mu^*(i_\mu(A)) =$ { a, c, d, e }, $\mathbf{A \not\subseteq i_\mu(c_\mu^*(i_\mu(A)))}$. Hence **A is not α-$\mathcal{H}$-open**. Therefore, A is β-$\mathcal{H}$-open but not α-$\mathcal{H}$-open. Hence every β-$\mathcal{H}$-open set need not be a α-$\mathcal{H}$-open.

Proposition 2.4 Every regular $\mathcal{H}$-open set is π-$\mathcal{H}$-open.

Remark 2.4 The converse of Proposition 2.4 need not be true as shown in Example 2.10.

Example 2.10 Let X = { a, b, c, d, e }, μ = { ϕ, { a }, { c }, { a, c }, { a, b, c }, { c, d }, { a, c, d }, { a, b, c, d } } and $\mathcal{H}$ = { ϕ, { a }, { b }, { c } }. Clearly, μ is a generalized topology and $\mathcal{H}$ is a hereditary class and the triple (X, μ, $\mathcal{H}$) is a hereditary generalized topological space. Let A = { a, c } be a subset of X. Then, $A^* =$ { a, b, c, d } and so $c_\mu^*(A) = A \cup A^* =$ { a, b, c, d}. Then, $i_\mu(c_\mu^*(A)) =$ { a, b, c, d }. Hence, $\mathbf{A \subset i_\mu(c_\mu^*(A))}$. Therefore, **A is π-$\mathcal{H}$-open**. But $i_\mu(c_\mu^*(A)) =$ { a, b, c, d }, $\mathbf{A \neq i_\mu(c_\mu^*(A))}$. Hence **A is not regular-$\mathcal{H}$-open**. Therefore, A is π-$\mathcal{H}$-open but not regular-$\mathcal{H}$-open. Hence every π-$\mathcal{H}$-open set need not be a regular-$\mathcal{H}$-open.

Proposition 2.5 Every regular-$\mathcal{H}$-open set is β-$\mathcal{H}$-open.

Remark 2.5 The converse of Proposition 2.5 need not be true as shown in Example 2.11.

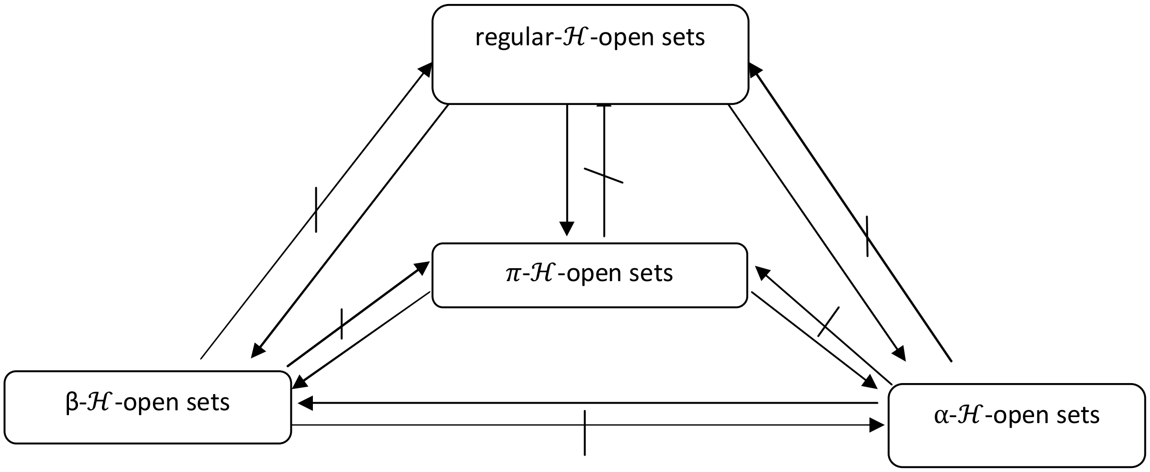
Example 2.11 Let X = { a, b, c, d, e, f }, μ = { ϕ, { d, e }, { e }, { a, c, e }, { a, c }, { a, c, d, e } } and $\mathcal{H}$ = { ϕ, { a } }. Clearly, μ is a generalized topology and $\mathcal{H}$ is a hereditary class and the triple (X, μ, $\mathcal{H}$) is a hereditary generalized topological space. The μ-closed sets are X, { a, b, c, f }, { a, b, c, d, f }, { b, d, f }, { b, d, e, f } and { b, f }. Let A = { a, c, f } be a subset of X. Then, $A^* =$ { a, c, d, e } and so $c_\mu^*(A) = A \cup A^* =$ { a, c, d, e, f }, $i_\mu(c_\mu^*(A)) =$ { a, c, d, e }. Thus, $c_\mu(i_\mu(c_\mu^*(A))) = X$. Hence, $\mathbf{A \subset c_\mu(i_\mu(c_\mu^*(A)))}$. Therefore, **A is β-$\mathcal{H}$-open**. But $i_\mu(c_\mu^*(A)) =$ { a, c, d, e }, $\mathbf{A \neq i_\mu(c_\mu^*(A))}$. Hence **A is not regular-$\mathcal{H}$-open**. Therefore, A is β-$\mathcal{H}$-open but not regular-$\mathcal{H}$-open. Hence every β-$\mathcal{H}$-open set need not be a regular-$\mathcal{H}$-open.

Proposition 2.6 Every regular-$\mathcal{H}$-open set is α-$\mathcal{H}$-open.

Remark 2.6 The converse of Proposition 2.6 need not be true as shown in Example 2.12.

Example 2.12 Let X = { a, b, c, d, e }, μ = { ϕ, { a }, { c }, { a, c }, { a, b, c }, { c, d }, { a, c, d }, { a, b, c, d } } and $\mathcal{H}$ = { ϕ, { a }, { b }, { c } }. Clearly, μ is a generalized topology and $\mathcal{H}$ is a hereditary class and the triple (X, μ, $\mathcal{H}$) is a hereditary generalized topological space. Let A = { a, c } be a subset of X. Then, $i_\mu(A) =$ { a, c } and so $c_\mu^*(i_\mu(A))$ = { a, b, c, d }. Then, $i_\mu(c_\mu^*(i_\mu(A)))$ = { a, b, c, d }. Hence, $\mathbf{A \subset i_\mu(c_\mu^*(i_\mu(A)))}$. Therefore, **A is α-$\mathcal{H}$-open**. But $i_\mu(c_\mu^*(A)) =$ { a, b, c, d }, $\mathbf{A \neq i_\mu(c_\mu^*(A))}$. Hence **A is not regular-$\mathcal{H}$-open**. Therefore, A is α-$\mathcal{H}$-open but not regular-$\mathcal{H}$-open. Hence every α-$\mathcal{H}$-open set need not be a regular-$\mathcal{H}$-open.

Remark 2.7 From the above discussions the following implications hold:



3. REGULAR-$\mathcal{H}$-CONTINUOUS FUNCTIONS

Definition 3.1 Let (X, μ_1, $\mathcal{H}_1$) and (Y, μ_2, $\mathcal{H}_2$) be any two hereditary generalized topological spaces. Any function f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$) is said to be a regular- $\mathcal{H}$-continuous function if $f^{-1}(A)$ is a regular-$\mathcal{H}$-open set in (X, μ_1, $\mathcal{H}_1$) for every μ_2-open set A of (Y, μ_2, $\mathcal{H}_2$).

Definition 3.2 Let (X, μ_1, $\mathcal{H}_1$) and (Y, μ_2, $\mathcal{H}_2$) be any two hereditary generalized topological spaces. Any function f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$) is said to be a μ-continuous function if f^{-1}(A) is a μ-open set in (X, μ_1, $\mathcal{H}_1$) for every μ_2-open set A of (Y, μ_2, $\mathcal{H}_2$).

Definition 3.3 Let (X, μ_1, $\mathcal{H}_1$) and (Y, μ_2, $\mathcal{H}_2$) be any two hereditary generalized topological spaces. Any function f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$) is said to be a β-$\mathcal{H}$-continuous function if f^{-1}(A) is a β-$\mathcal{H}$-open set in (X, μ_1, $\mathcal{H}_1$) for every μ_2-open set A of (Y, μ_2, $\mathcal{H}_2$).

Definition 3.4 Let (X, μ_1, $\mathcal{H}_1$) and (Y, μ_2, $\mathcal{H}_2$) be any two hereditary generalized topological spaces. Any function f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$) is said to be a π-$\mathcal{H}$-continuous function if f^{-1}(A) is a π-$\mathcal{H}$-open set in (X, μ_1, $\mathcal{H}_1$) for every μ_2-open set A of (Y, μ_2, $\mathcal{H}_2$).

Definition 3.5 Let (X, μ_1, $\mathcal{H}_1$) and (Y, μ_2, $\mathcal{H}_2$) be any two hereditary generalized topological spaces. Any function f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$) is said to be an α-$\mathcal{H}$-continuous function if the set f^{-1}(A) is an α-$\mathcal{H}$-open set in (X, μ_1, $\mathcal{H}_1$) for every μ_2-open set A of (Y, μ_2, $\mathcal{H}_2$).

Proposition 3.1 Let (X, μ_1, $\mathcal{H}_1$) and (Y, μ_2, $\mathcal{H}_2$) be any two hereditary generalized topological spaces. Then for any function f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$), the following statements are equivalent:

(a) f is a regular-$\mathcal{H}$-continuous function

(b) For every subset A of X, $f(R_{\mathcal{H}}cl(A)) \subseteq c_\mu(f(A))$

(c) For every subset B of Y, $R_{\mathcal{H}}cl(f^{-1}(B)) \subseteq f^{-1}(c_\mu(B))$

(d) For every subset B of Y, $f^{-1}(i_\mu(B)) \subseteq R_{\mathcal{H}}int(f^{-1}(B))$.

Proof: **(a) ⇒ (b)** For any subset A of X, Y □ $c_\mu(f(A))$ is μ_2-open. Since f is a regular-$\mathcal{H}$-continuous function, $f^{-1}(Y - c_\mu(f(A))$ is regular $\mathcal{H}$ open and so $f^{-1}(c_\mu(f(A))$ is regular-$\mathcal{H}$-closed. Since $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(c_\mu(f(A))$, it follows that $R_{\mathcal{H}}cl(A) \subseteq R_{\mathcal{H}}cl(f^{-1}(c_\mu(f(A))) \subseteq f^{-1}(c_\mu(f(A)))$. Hence, $f(R_{\mathcal{H}}cl(A)) \subseteq c_\mu(f(A))$.

(b) ⇒ (c) Let B be any subset of Y. By (b), $f(R_{\mathcal{H}}cl(f^{-1}(B)) \subseteq c_\mu(f(f^{-1}(B))) \subseteq c_\mu(B)$. Hence $R_{\mathcal{H}}cl(f^{-1}(B)) \subseteq (f^{-1}(c_\mu(B))$.

(c) ⇒ (d) Let B be any subset of Y. By (c), $f^{-1}(c_\mu(Y$ □ $B)) \supseteq R_{\mathcal{H}}cl(f^{-1}(Y$ □ $B)) = R_{\mathcal{H}}cl(X$ □ $f^{-1}(B))$. Hence, $R_{\mathcal{H}}int(f^{-1}(B)) \supseteq (f^{-1}(i_\mu(B)))$.

(d) ⇒ (a) Let A be any μ_2-open set of Y. Then, $i_\mu(A) = A$. Now, $f^{-1}(A)) = f^{-1}(i_\mu(A)) \subseteq R_{\mathcal{H}}int(f^{-1}(A))$. Thus, $f^{-1}(A)) \subseteq R_{\mathcal{H}}int(f^{-1}(A))$. It is always true that $R_{\mathcal{H}}int(f^{-1}(A)) \subseteq f^{-1}(A))$. Thus $R_{\mathcal{H}}int(f^{-1}(A)) = f^{-1}(A)$. Hence $f^{-1}(A)$ is regular-$\mathcal{H}$-open, which implies that f is a regular-$\mathcal{H}$-continuous function.

Definition 3.6 Let $(X, \mu_1, \mathcal{H}_1)$ and $(Y, \mu_2, \mathcal{H}_2)$ be any two hereditary generalized topological spaces. Any function $f : (X, \mu_1, \mathcal{H}_1) \rightarrow (Y, \mu_2, \mathcal{H}_2)$ is said to be a regular-$\mathcal{H}$-irresolute if $f^{-1}(V)$ is regular-$\mathcal{H}$-open in $(X, \mu_1, \mathcal{H}_1)$ for every regular-$\mathcal{H}$-open set V in $(Y, \mu_2, \mathcal{H}_2)$.

Proposition 3.2 Let $(X, \mu_1, \mathcal{H}_1)$ and $(Y, \mu_2, \mathcal{H}_2)$ be any two hereditary generalized topological spaces. Then for any function $f : (X, \mu_1, \mathcal{H}_1) \rightarrow (Y, \mu_2, \mathcal{H}_2)$, the following statements are equivalent:

(a) f is a regular-$\mathcal{H}$- irresolute function.

(b) $f(R_{\mathcal{H}}cl(A)) \subseteq R_{\mathcal{H}}cl(f(A))$ for every subset A of X.

(c) $R_{\mathcal{H}}cl(f^{-1}(B)) \subseteq f^{-1}(R_{\mathcal{H}}cl(B))$ for every subset B of Y.

Proof: **(a) ⇒ (b)** Let f be a regular-$\mathcal{H}$-irresolute function. Let $A \subseteq X$. Then $R_{\mathcal{H}}cl(f(A))$ is a regular-$\mathcal{H}$-closed set in $(Y, \mu_2, \mathcal{H}_2)$. By (a), $f^{-1}(R_{\mathcal{H}}cl(f(A))$ is regular-$\mathcal{H}$-closed in $(X, \mu_1, \mathcal{H}_1)$. Now, $A \subseteq f^{-1}(f(A))$. Thus, $R_{\mathcal{H}}cl(A) \subseteq R_{\mathcal{H}}cl(f^{-1}(f(A))) \subseteq R_{\mathcal{H}}cl(f^{-1}(R_{\mathcal{H}}cl(f(A)))) = f^{-1}(R_{\mathcal{H}}cl(f(A)))$. Hence, $f(R_{\mathcal{H}}cl(A)) \subseteq R_{\mathcal{H}}cl(f(A))$.

(b) ⇒ (c) Let $B \in Y$ then $f^{-1}(B) \in X$. By (b), $f(R_{\mathcal{H}}cl(f^{-1}(B))) \subseteq R_{\mathcal{H}}cl(f(f^{-1}(B))) \subseteq R_{\mathcal{H}}cl(B)$. Thus, $f^{-1}(f(R_{\mathcal{H}}cl(f^{-1}(B)))) \subseteq f^{-1}(R_{\mathcal{H}}cl(B))$. That is, $R_{\mathcal{H}}cl(f^{-1}(B)) \subseteq f^{-1}(R_{\mathcal{H}}cl(B))$.

(c) ⇒ (a) Let $S \in Y$ be a regular $\mathcal{H}$ closed set. Then, $R_{\mathcal{H}}cl(S) = S$. By (c), it follows that $R_{\mathcal{H}}cl(f^{-1}(S)) \subseteq f^{-1}(R_{\mathcal{H}}cl(S)) = f^{-1}(S)$. But, $f^{-1}(S) \subseteq R_{\mathcal{H}}cl(f^{-1}(S))$. Therefore, $f^{-1}(S) = R_{\mathcal{H}}cl(f^{-1}(S))$. Hence $f^{-1}(S)$ is a regular $\mathcal{H}$ closed set in X. Thus f is a regular-$\mathcal{H}$-irresolute function.

Definition 3.7 Let $(X, \mu_1, \mathcal{H}_1)$ and $(Y, \mu_2, \mathcal{H}_2)$ be any two hereditary generalized topological spaces. Any function $f : (X, \mu_1, \mathcal{H}_1) \rightarrow (Y, \mu_2, \mathcal{H}_2)$ is said to be a μ-closed function (resp. μ-open function) if f(F) is a μ-closed (resp. μ-open) set in $(Y, \mu_2, \mathcal{H}_2)$ for every μ_1-closed set F of $(X, \mu_1, \mathcal{H}_1)$.

Definition 3.8 Let $(X, \mu_1, \mathcal{H}_1)$ and $(Y, \mu_2, \mathcal{H}_2)$ be any two hereditary generalized topological spaces. Any function $f : (X, \mu_1, \mathcal{H}_1) \rightarrow (Y, \mu_2, \mathcal{H}_2)$ is said to be a $R_{\mathcal{H}}$-closed function (resp.

$R_{\mathcal{H}}$-open function) if f(F) is a $R_{\mathcal{H}}$-closed (resp. $R_{\mathcal{H}}$-open) set in (Y, μ_2, $\mathcal{H}_2$) for every μ_1-closed set F of (X, μ_1, $\mathcal{H}_1$).

Theorem 3.1 Let (X, μ_1, $\mathcal{H}_1$) and (Y, μ_2, $\mathcal{H}_2$) be any two hereditary generalized topological spaces. Then for any bijective function f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$), the following statements are equivalent:

(a) f is $R_{\mathcal{H}}$-open,

(b) f is $R_{\mathcal{H}}$-closed,

(c) f^{-1} is $R_{\mathcal{H}}$-irresolute.

Proof: **(a) ⇒ (b)** Suppose f is $R_{\mathcal{H}}$-open. Let F be $R_{\mathcal{H}}$-closed in X. Then X\F is $R_{\mathcal{H}}$-open. By Definition 3.7, f(X\F) is $R_{\mathcal{H}}$-open. Since f is bijection, f(X\F) = Y\f(F) is $R_{\mathcal{H}}$-open in Y. Hence f(F) is $R_{\mathcal{H}}$-closed. Therefore f is $R_{\mathcal{H}}$-closed.

(b) ⇒ (c) Let g = f^{-1}. Suppose f is $R_{\mathcal{H}}$-closed. Let V be $R_{\mathcal{H}}$-open in X. Then X\V is $R_{\mathcal{H}}$-closed in X. Since f is $R_{\mathcal{H}}$-closed, f(X\V) is $R_{\mathcal{H}}$-closed. Since f is a bijection, Y\f(V) is $R_{\mathcal{H}}$-closed which implies that f(V) is $R_{\mathcal{H}}$-open in Y. Since g = f^{-1} and since g and f are bijection g^{-1} (V) = f(V) so that g^{-1}(V) is $R_{\mathcal{H}}$-open in Y. Therefore f^{-1} is $R_{\mathcal{H}}$-irresolute.

(c) ⇒ (a) Suppose f^{-1} is $R_{\mathcal{H}}$-irresolute. Let V be $R_{\mathcal{H}}$-open in X. Then X\V is $R_{\mathcal{H}}$-closed in X. Since f^{-1} is $R_{\mathcal{H}}$-irresolute and $(f^{-1})^{-1}$ (X\V) = f(X\V) = Y\f(V) is $R_{\mathcal{H}}$-closed in Y which implies that f(V) is $R_{\mathcal{H}}$-open in Y. Therefore f is $R_{\mathcal{H}}$-open.

Theorem 3.2 Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space and let A be any subset of X. Then x ∈ $R_{\mathcal{H}}cl(A)$ if and only if for any regular-$\mathcal{H}$-open set U containing x, A ∩ U ≠ ϕ.

Proof: Let x ∈ $R_{\mathcal{H}}cl(A)$ and suppose that, there is a regular-$\mathcal{H}$-open set U in (X, μ, $\mathcal{H}$) such that x ∈ U and A ∩ U = ϕ. This implies that A ⊂ U^c (where U^c = X \U) which is regular-$\mathcal{H}$-closed in (X, μ, $\mathcal{H}$). Hence $R_{\mathcal{H}}cl(A) \subseteq R_{\mathcal{H}}cl(U^c) = U^c$. since x ∈ U implies that x ∉ U^c implies that x ∉ $R_{\mathcal{H}}cl(A)$, which is a contradiction. Hence if x ∈ $R_{\mathcal{H}}cl(A)$ then there is a regular-$\mathcal{H}$-open set U with x ∈ U such that A ∩ U ≠ ϕ.

Conversely, assume that, for any regular-$\mathcal{H}$-open set U containing x, A ∩ U ≠ ϕ. To prove that x ∈ $R_{\mathcal{H}}cl(A)$. Suppose that x ∉ $R_{\mathcal{H}}cl(A)$, then there is a regular-$\mathcal{H}$-closed set F in (X, μ, $\mathcal{H}$) such that x ∉ F and A ⊆ F. Since x ∉ F implies that x ∈ F^c which is regular-$\mathcal{H}$-open in (X, μ, $\mathcal{H}$). Since A ⊆ F implies that A ∩ F^c = ϕ, which is a contradiction. Thus x ∈ $R_{\mathcal{H}}cl(A)$.

Theorem 3.3 Let (X, μ_1, $\mathcal{H}_1$), (Y, μ_2, $\mathcal{H}_2$) and (Z, μ_3, $\mathcal{H}_3$) be any three hereditary generalized topological spaces. Let f: (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$) and g : (Y, μ_2, $\mathcal{H}_2$) → (Z, μ_3, $\mathcal{H}_3$) be any two functions. Let h = g ∘ f. Then:

(a) h is regular-$\mathcal{H}$-continuous if f is regular-$\mathcal{H}$-irresolute and g is regular-$\mathcal{H}$-continuous

(b) h is regular-$\mathcal{H}$-continuous if g is μ-continuous and f is regular-$\mathcal{H}$-continuous.

Proof:

(a) Let V be μ_3-closed in (Z, μ_3, $\mathcal{H}_3$). Suppose f is regular-$\mathcal{H}$-irresolute and g is regular-$\mathcal{H}$-continuous. Since g is regular-$\mathcal{H}$-continuous, g^{-1}(V) is regular-$\mathcal{H}$-closed in (Y, μ_2, $\mathcal{H}_2$). Since f is regular-$\mathcal{H}$-irresolute, $f^{-1}(g^{-1}(V))$ is regular-$\mathcal{H}$-closed in (X, μ_1, $\mathcal{H}_1$). Hence h is regular-$\mathcal{H}$-continuous function.

(b) Let V be closed in (Z, μ_3, $\mathcal{H}_3$). Suppose g is μ-continuous and f is regular-$\mathcal{H}$-continuous. Then g^{-1}(V) is μ-closed in (Y, μ_2, $\mathcal{H}_2$). Since f is regular-$\mathcal{H}$-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V) = h^{-1}(V)$ is regular-$\mathcal{H}$-closed in (X, μ_1, $\mathcal{H}_1$). Hence h is regular-$\mathcal{H}$-continuous function.

Theorem 3.4 Let (X, μ_1, $\mathcal{H}_1$), (Y, μ_2, $\mathcal{H}_2$) and (Z, μ_3, $\mathcal{H}_3$) be any three hereditary generalized topological spaces. Let f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$) and g : (Y, μ_2, $\mathcal{H}_2$) → (Z, μ_3, $\mathcal{H}_3$) are regular-$\mathcal{H}$-irresolute functions, then g ∘ f : (X, μ_1, $\mathcal{H}_1$) → (Z, μ_3, $\mathcal{H}_3$) is regular-$\mathcal{H}$-irresolute function.

Proof: Let g be an regular-$\mathcal{H}$-irresolute function and V be any regular-$\mathcal{H}$-open in (Z, μ_3, $\mathcal{H}_3$), then f^{-1}(V) is regular-$\mathcal{H}$-open set in Y, since f is regular-$\mathcal{H}$-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V) = h^{-1}(V)$ is regular-$\mathcal{H}$-open in (X, μ_1, $\mathcal{H}_1$). Hence g ∘ f is regular-$\mathcal{H}$-irresolute.

Proposition 3.3 Every π-$\mathcal{H}$-continuous function is a β-$\mathcal{H}$-continuous function.

Remark 3.1 The converse of Proposition 3.3 need not be true as shown in Example 3.7.

Example 3.7 Let X = { a, b, c, d, e, f }, Y = { x, y, z }. Let μ_1 = { ϕ, { d, e }, { e }, { a, c, e }, { a, c }, { a, c, d, e } }and μ_2 = { ϕ, { x } }. Clearly, μ_1 and μ_2 are generalized topologies on X and Y respectively. Let $\mathcal{H}_1$ = { ϕ, { a } } and $\mathcal{H}_2$ = { ϕ, { x } } are hereditary classes on X and Y respectively. Then the triples (X, μ_1, $\mathcal{H}_1$) and (Y, μ_2, $\mathcal{H}_2$) are hereditary generalized topological spaces. Let f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$) be defined by f(a) = x, f(b) = y, f(c) = x, f(d) = y, f(e) = z and f(f) = x. For A = { x } ∈ μ_2, f^{-1}(A) = { a, c, f } and $c_\mu(i_\mu(c^*_\mu(f^{-1}(A))))$ = X and $f^{-1}(A) \subset c_\mu(i_\mu(c^*_\mu(f^{-1}(A))))$. Hence f^{-1}(A) is β- $\mathcal{H}$-open in (X, μ_1, $\mathcal{H}_1$). Similarly, for A = ϕ, $f^{-1}(\phi) = \phi$ which is a β-$\mathcal{H}$-open in (X, μ_1, $\mathcal{H}_1$). Hence, the inverse image of every μ_2-open set in (Y, μ_2, $\mathcal{H}_2$) is β-$\mathcal{H}$-open set in (X, μ_1, $\mathcal{H}_1$). But $f^{-1}(A) \nsubseteq i_\mu(c^*_\mu(f^{-1}(A)))$ as $i_\mu(c^*_\mu(f^{-1}(A)))$ = { a, c, d, e }. Thus f^{-1}(A) is not π-$\mathcal{H}$-open. Thus f is no`t π- $\mathcal{H}$-continuous function. Hence f is a β-$\mathcal{H}$-continuous function but not π- $\mathcal{H}$-continuous function. Hence every β-$\mathcal{H}$-continuous function need not be a π- $\mathcal{H}$-continuous function.

Proposition 3.4 Every α-$\mathcal{H}$-continuous function is a π-$\mathcal{H}$-continuous.

Remark 3.2 The converse of Proposition 3.4 need not be true as shown in Example 3.8.

Example 3.8 Let X = { a, b, c, d, e }, Y = { x, y, z }. Let μ_1 = { ϕ, { a }, { c }, { a, c }, { a, b, c }, { c, d }, { a, c, d }, { a, b, c, d } } and μ_2 = { ϕ, { x } }. Clearly, μ_1 and μ_2 are generalized topologies on X and Y respectively. Let $\mathcal{H}_1$ = { ϕ, { a }, { b }, { c } } and $\mathcal{H}_2$ = { ϕ, { x } } hereditary classes on X and Y respectively. Then the triples (X, μ_1, $\mathcal{H}_1$) and (Y, μ_2, $\mathcal{H}_2$) are hereditary generalized topological spaces. Let f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$) be defined by f(a) = x, f(b) = x, f(c) = y, f(d) = x and f(e) = z. For A = { x } ∈ μ_2, f^{-1}(A) = { a, b, d } and $i_\mu(c^*_\mu(f^{-1}(A)))$ = { a, b, c, d } and f^{-1}(A) ⊂ $i_\mu(c^*_\mu(f^{-1}A))$. Hence f^{-1}(A) is π-$\mathcal{H}$-open in (X, μ_1, $\mathcal{H}_1$). Similarly, for A = ϕ, f^{-1}(ϕ) = ϕ which is a π-$\mathcal{H}$-open in (X, μ_1, $\mathcal{H}_1$). Hence, the inverse image of every μ_2-open set in (Y, μ_2, $\mathcal{H}_2$) is π-$\mathcal{H}$-open set in (X, μ_1, $\mathcal{H}_1$). But f^{-1}(A) $\nsubseteq i_\mu(c^*_\mu(i_\mu(f^{-1}(A))))$ as $i_\mu(c^*_\mu(i_\mu(f^{-1}(A))))$ = { a }. Thus f^{-1}(A) is not α-$\mathcal{H}$-open. Therefore f is not α-$\mathcal{H}$-continuous function. Hence f is a π-$\mathcal{H}$-continuous function but not α-$\mathcal{H}$-continuous function. Hence every π-$\mathcal{H}$-continuous function need not be α-$\mathcal{H}$-continuous function.

Proposition 3.5 Every α-$\mathcal{H}$-continuous function is a β-$\mathcal{H}$-continuous function.

Remark 3.3 The converse of Proposition 3.5 need not be true as shown in Example 3.9.

Example 3.9 Let X = { a, b, c, d, e, f }, Y = { x, y, z }. Let μ_1 = { ϕ, { d, e }, { e }, { a, c, e }, { a, c }, { a, c, d, e } }and μ_2 = { ϕ, { x } }. Clearly, μ_1 and μ_2 are generalized topologies on X and Y respectively. Let $\mathcal{H}_1$ = { ϕ, { a } } and $\mathcal{H}_2$ = { ϕ, { x } } be hereditary classes on X and Y respectively. Then the triples (X, μ_1, $\mathcal{H}_1$) and (Y, μ_2, $\mathcal{H}_2$) are hereditary generalized topological spaces. Let f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$) be defined by f(a) = x, f(b) = y, f(c) = x, f(d) = y, f(e) = z and f(f) = x. For A = { x } ∈ μ_2, f^{-1}(A) = { a, c, f } and $c_\mu(i_\mu(c^*_\mu(f^{-1}(A))))$ = X and $f^{-1}(A) \subset c_\mu(i_\mu(c^*_\mu(f^{-1}(A))))$. Hence f^{-1}(A) is β-$\mathcal{H}$-open in (X, μ_1, $\mathcal{H}_1$). Similarly, for A = ϕ, f^{-1}(ϕ) = ϕ which is a β-$\mathcal{H}$-open in (X, μ_1, $\mathcal{H}_1$). Hence, the inverse image of every μ_2-open set in (Y, μ_2, $\mathcal{H}_2$) is β-$\mathcal{H}$-open set in (X, μ_1, $\mathcal{H}_1$). But f^{-1}(A) $\nsubseteq i_\mu(c^*_\mu(i_\mu(f^{-1}(A))))$ as $i_\mu(c^*_\mu(i_\mu(f^{-1}(A))))$ = { a, c, d, e }. Thus f^{-1}(A) is not α-$\mathcal{H}$-open. Therefore f is not α-$\mathcal{H}$-continuous function. Hence f is a β-$\mathcal{H}$-continuous function but not α-$\mathcal{H}$-continuous function. Hence every β-$\mathcal{H}$-continuous function need not be an α-$\mathcal{H}$-continuous function.

Proposition 3.6 Every regular-$\mathcal{H}$-continuous function is a π-$\mathcal{H}$-continuous function.

Remark 3.4 The converse of Proposition 3.6 need not be true as shown in Example 3.10.

Example 3.10 Let X = { a, b, c, d, e, f }, Y = { x, y, z }. Let μ_1 = { ϕ, { d, e }, { e }, { a, c, e }, { a, c }, { a, c, d, e } }and μ_2 = { ϕ, { x } }. Clearly, μ_1 and μ_2 are generalized topologies on X and Y respectively. Let $\mathcal{H}_1$ = { ϕ, { a } } and $\mathcal{H}_2$ = { ϕ, { x } } are hereditary classes on X and Y and the triples (X, μ_1, $\mathcal{H}_1$) and (Y, μ_2, $\mathcal{H}_2$) are hereditary generalized topological spaces. Let f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$) be defined by f(a) = x, f(b) = y, f(c) = x, f(d) = y, f(e) = z and f(f) = x. For A = {x} ∈ μ_2, f^{-1}(A) = { a, c, f } and $i_\mu(c^*_\mu(f^{-1}(A)))$ = { a, b, c, d } and f^{-1}(A) ⊂ $i_\mu(c^*_\mu(f^{-1}(A)))$. Hence f^{-1}(A) is π-$\mathcal{H}$-open in (X, μ_1, $\mathcal{H}_1$). Similarly, for A = ϕ, f^{-1}(ϕ) = ϕ which is a π-$\mathcal{H}$-open in (X, μ_1, $\mathcal{H}_1$). Hence, the inverse image of every μ_2-open set in (Y, μ_2, $\mathcal{H}_2$) is π-$\mathcal{H}$-open set in (X, μ_1, $\mathcal{H}_1$). But $f^{-1}(A) \neq i_\mu(c^*_\mu(f^{-1}(A)))$ as $i_\mu(c^*_\mu(f^{-1}(A)))$ = { a, b, c, d }. Thus f^{-1}(A) is not regular-$\mathcal{H}$-open. Therefore f is not regular-$\mathcal{H}$-continuous function. Hence f is a π-$\mathcal{H}$-continuous function but not a regular- $\mathcal{H}$-continuous function. Hence every π-$\mathcal{H}$-continuous function need not be a regular- $\mathcal{H}$-continuous function.

Proposition 3.7 Every regular-$\mathcal{H}$-continuous function is a β-$\mathcal{H}$-continuous function.

Remark 3.5 The converse of Proposition 3.7 need not be true as shown in Example 3.11.

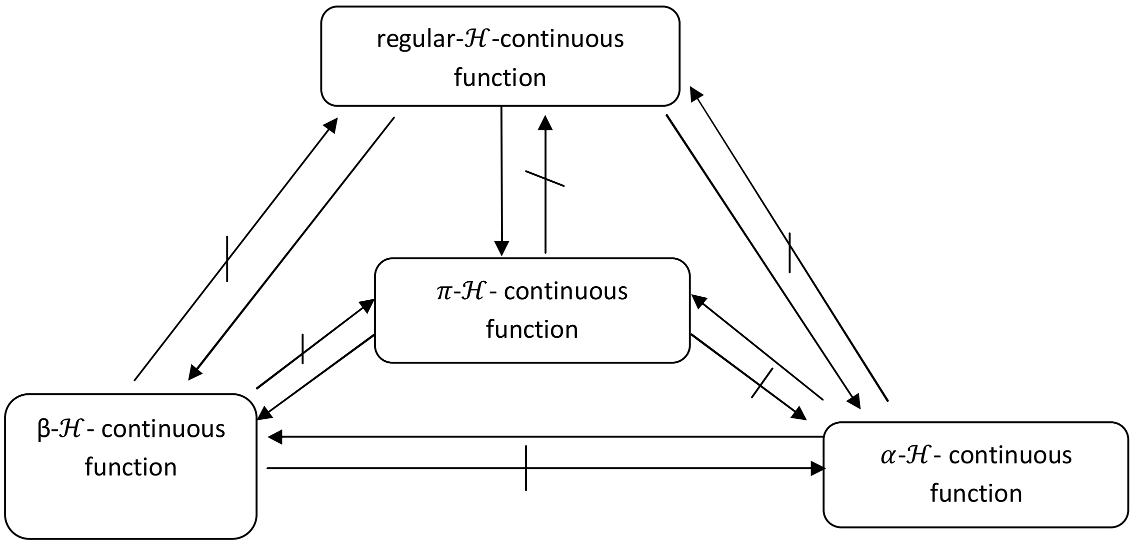
Example 3.11 Let X = { a, b, c, d, e }, Y = { x, y, z }. Let μ_1 = { ϕ, { a }, { c }, { a, c }, { a, b, c }, { c, d }, { a, c, d }, { a, b, c, d } } and μ_2 = { ϕ, { x } }. Clearly, μ_1 and μ_2 are generalized topologies on X and Y respectively. Let $\mathcal{H}_1$ = { ϕ, { a }, { b }, { c } } and $\mathcal{H}_2$ = { ϕ, { x } }hereditary classes on X and Y and the triples (X, μ_1, $\mathcal{H}_1$) and (Y, μ_2, $\mathcal{H}_2$) are hereditary generalized topological spaces. Let f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$) be defined by f(a) = x, f(b) = y, f(c) = x, f(d) = y and f(e) = z. For A = { x } ∈ μ_2, f^{-1}(A) = { a, c } and $c_\mu(i_\mu(c^*_\mu(f^{-1}(A))))$ = X and f^{-1}(A) ⊂ $c_\mu(i_\mu(c^*_\mu(f^{-1}(A))))$. Hence f^{-1}(A) is β- $\mathcal{H}$-open in (X, μ_1, $\mathcal{H}_1$). Similarly, for A = ϕ, f^{-1}(ϕ) = ϕ which is an α- $\mathcal{H}$-open in (X, μ_1, $\mathcal{H}_1$). Hence, the inverse image of every μ-open set in (Y, μ_1, $\mathcal{H}_1$) is β-$\mathcal{H}$-open set in (X, μ_1, $\mathcal{H}_1$). But f^{-1}(A) ≠ $i_\mu(c^*_\mu(f^{-1}(A)))$ as $i_\mu(c^*_\mu(f^{-1}(A)))$ = { a, c, d, e }.Thus f^{-1}(A) is not regular-$\mathcal{H}$-open. Therefore f is not regular-$\mathcal{H}$-continuous function. Hence f is a β-$\mathcal{H}$-continuous function but not a regular- $\mathcal{H}$-continuous function. Hence every β-$\mathcal{H}$-continuous function need not be a regular- $\mathcal{H}$-continuous function.

Proposition 3.8 Every regular-$\mathcal{H}$-continuous function is an α-$\mathcal{H}$-continuous function.

Remark 3.6 The converse of Proposition 3.8 need not be true as shown in Example 3.12.

Example 3.12 Let X = { a, b, c, d, e }, Y = { x, y, z }. Let μ_1 = { ϕ, { a }, { c }, { a, c }, { a, b, c }, { c, d }, { a, c, d }, { a, b, c, d } } and μ_2 = { ϕ, { x } }. Clearly, μ_1 and μ_2 are generalized topologies on X and Y respectively. Let $\mathcal{H}_1$ = { ϕ, { a }, { b }, { c } } and $\mathcal{H}_2$ = { ϕ, { x } } are hereditary classes on X and Y and the triples (X, μ_1, $\mathcal{H}_1$) and (Y, μ_2, $\mathcal{H}_2$) are hereditary generalized topological spaces. Let f : (X, μ_1, $\mathcal{H}_1$) → (Y, μ_2, $\mathcal{H}_2$) be defined by f(a) = x, f(b) = y, f(c) = x, f(d) = y and f(e) = z. For A = { x } ∈ μ_2, f^{-1}(A) = { a, c } and $i_\mu(c^*_\mu(i_\mu(f^{-1}(A))))$ = { a, b, c, d } and $f^{-1}(A) \subset i_\mu(c^*_\mu(i_\mu(f^{-1}(A))))$. Hence f^{-1}(A) is an α-$\mathcal{H}$-open in (X, μ_1, $\mathcal{H}_1$). Similarly, for A = ϕ, f^{-1}(ϕ) = ϕ which is an α- $\mathcal{H}$-open in (X, μ_1, $\mathcal{H}_1$). Hence, the inverse image of every μ_2-open set in (Y, μ_1, $\mathcal{H}_1$) is an α-$\mathcal{H}$-open set in (X, μ_1, $\mathcal{H}_1$). But f^{-1}(A) ≠ $i_\mu(c^*_\mu(f^{-1}(A)))$ as $i_\mu(c^*_\mu(f^{-1}(A)))$ = { a, b, c, d }.Thus f^{-1}(A) is not regular-$\mathcal{H}$-open. Therefore f is not regular-$\mathcal{H}$-continuous function. Hence f is an α-$\mathcal{H}$-continuous function but not a regular-$\mathcal{H}$-continuous function. Hence every α-$\mathcal{H}$-continuous function need not be a regular-$\mathcal{H}$-continuous function.

Remark 3.7 From the above discussions the following implications hold:



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