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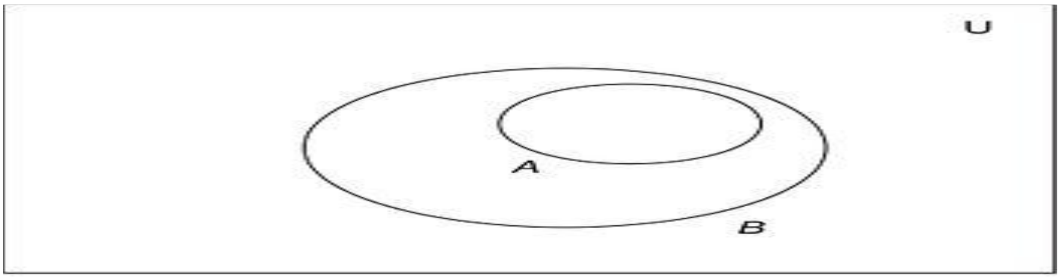

$A \subset B$

B.Amudhambigai

A STUDY ON SUPRA Z-PRECONTINUOUS FUNCTION

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Abstract

In this paper the concepts of supra zero sets, supra co-zero sets, supra coprezero sets are introduced and their properties are studied. Also, this paper gives a brief survey on supra Z-precontinuous functions and their properties.

Keywords : supra zero sets, supra co-zero sets, supra coprezero sets and supra Z-precontinuous functions. 2000 Mathematics Subject Classification. Primary 54D10, 54D15.

1.INTRODUCTION AND PRELIMINARIES

Topology is a type of geometry which deals with the properties of figures and surfaces that remain unchanged during stretching, bending, contracting and twisting operations. Instead of dealing with magnitudes of lengths and angles it deals with topological properties of figures and surfaces. Topological concepts are used in design of networks meant for the distribution of electricity, gas and water and for designing industrial automation. They are used in the control of automobile traffic and guided missiles. They are also applied to the designing of geographical maps.

The concept of Pre-open set was introduced by Mashhoru A.S., Abd El- Monsef M.E. and El-Deeb S.N. in [7]. The concept of Pre-continuity was discussed and studied by Gnanambal Y.

Navalagi [10] introduced the concepts of pre-zero sets and copre-zero sets of a space with the help of the precontinuous functions.

In 1983, Mashhour A.S. et al. [8] introduced the supra topological spaces and studied s-continuous functions and s*-continuous function.

This paper gives a brief survey on supra Z-precontinuous functions and its properties.

1.2 PRELIMINARIES

DEFINITION 1.2.5 [2]

Let X be a non-empty set. A family T of subsets of X is said to be topology on X if and only if T satisfies the following axioms:

a) ϕ and X are in T.
b) The union of the elements of any sub-collection of T is in T.
c) The finite intersection of the elements of any sub-collection of T is in T.

Then, T is a topology on X. The ordered (X, T) is called a topological space.

DEFINITION 1.2.6 [4]

Let x be a point in a topological space (X, T). A set U in X is said to be a neighborhood of x if there exists an open set G in X such that $x \in G \subseteq U$

DEFINITION 1.2.7 [4]

Let (X, T) be a topological space. Let A be a subset of (X,T). The union of all open sets contained in A is called the interior of A and it is denoted by Int(A) or $\bar{A}$.

DEFINITION 1.2.8 [4]

Let (X, T) be a topological space. Let A be a subset of (X, T). The intersection of all closed sets containing A is called the closure of A and it is denoted by Cl(A) or $\bar{A}$.
i.e., $Cl(A) = \cap \{ F/A \subseteq F, X - F \in T \}$

DEFINITION 1.2.11 [4]

Let (X, T) be a topological space and let Y be a subset of (X,T). The family $TY = \{ G \cap Y; G \in T \}$ satisfies the axioms of a topology of Y. The ordered pair (Y, TY) is called a subspace of X and the topology TY on Y is called the subspace topology on $Y \subset X$ and it is induced by the topology T on X.

DEFINITION 1.2.12 [6, 7]

A subset A of a topological space (X, T) is called pre-open if $A \subseteq Int(Cl(A))$. Clearly every open set in X as well as a dense set in X is pre-open. The complement of a pre-open set of (X, T) is called a pre-closed set. The family of all pre-open (pre-closed) sets of (X, T) is denoted by PO(X) (resp.PF(X)).

DEFINITION 1.2.13 [4]

A metric on a set X is a function $d : X \quad X \to R$ having the following properties:

1) $d(x, y) \geq 0$ for all $x, y \in X$; equality holds if and only if $x = y$.
2) $d(x, y) = d(y, x)$ for all $x, y \in X$.
3) (Triangle inequality) $d(x,y) + d(y,z) \geq d(x, z)$ for all $x,y, z \in X$.

DEFINITION 1.2.14 [4]

Let X be a topological space, X is said to be metrizable if there exists a metric d on the set X that induces the topology of X. A metric space is a metrizable space X together with a specific mertic d that gives the topology of X.

EXAMPLE 1.2.10

R is a metric space.

PROOF

A metric on a set R is a function d : $\mathrm{R} \times \mathrm{R} \rightarrow [\,0, \infty\,)$ defined by

d(x, x) = 0 (x ∈ R)............. (1)

d(x, y) = 1 (x, y ∈ R ; x □ y)............. (2)

1) Since by (2), d(x, y) = 1 □ 0 for all x, y∈ R.
   by (1), d(x, y) = 0 for x = y.
2) Since by (2), d(x, y) = 1 and d(y, x) = 1
   Therefore, d(x, y) = d(y, x) for all x, y ∈ R.
3) Since by (2), for all x, y, z ∈ R.
   Then, d(x, z) □ d(x, y) + d(y, z)
   1 < 1 + 1
   1 < 2
   Hence, triangle inequality is satisfied. Thus, R is a metric space.

DEFINITION 1.2.15 [10]

Let (X, T) be a topological space. A subset A of a space X is called a zero set if there exists a continuous functions f : (X, T) → R, where R is defined to be a neighborhood of a real number x, such that A
= { x ∈ X : f(x) = 0 }. The complement of a zero set of a space X is called a co-zero set C of X.

REMARK 1.2.1 [10]

Every closed set is a zero set.

DEFINITION 1.2.16 [5]

A subset A of a topological space X is called Z-open if for each x ∈ A, there exists a co-zero set C of X such that x ∈ C ⊂ A, or equivalently, if A can be expressed as the union of co-zero sets of X. The complement of a Z-open set is called Z-closed.

DEFINITION 1.2.17 [7]

Let (X, T) be a topological space. For any set A in X, the intersection of pre-closed sets containing the set A is called the pre-closure of A and we denote it by pCl(A) and pCl(A) is a pre-closed set.

i.e., Cl(A) = { B : B ⊇ A, B - pre-closed in X } is called pre-closure of A.

DEFINITION 1.2.18 [7]

Let (X, T) be a topological space. For any set A in X, the union of pre-open sets containing the set A is called the pre-interior of A and we denote it by pInt(A) and pInt(A) is a pre-open set.

i.e., Int(A) = ∪ { B : A ⊇ B, B - pre-open in X } is called pre-interior of A.

DEFINITION 1.2.19 [9]

A subset U of X containing a point $x \in X$ is called a pre-neighborhood (or pre-nbd) of x in X if there exists $A \in PO(X)$ such that $x \in A \subset U$.

REMARK 1.2.2 [9] Every pre-neighborhood A of x in X is pre-open in X.

DEFINITION 1.2.20 [7]

Let (X, T) and (Y, S) be any two topological spaces. A function $f : X \to Y$ is said to be pre-continuous at a point $x \in X$ if for every open set V of Y containing f(x) there exists a pre-open set U in X, such that $x \in U$ and $f(U) \subseteq V$. The function is said to be pre-continuous if it is pre-continuous at each $x \in X$.

DEFINITION 1.2.21 [10]

A subset A of a topological space X is said to be prezero set of X if there exists a precontinuous function $f : X \to R$ such that $A = \{ x \in X : f(x) = 0 \}$ and is denoted by PZ(f).

Its complement is called coprezero set of X. Every zero set is a prezero set.

DEFINITION 1.2.22 [11]

Let (X, T) and (Y, S) be any two topological spaces. A function $f : X \to Y$ is said to be Z-continuous if and only if the inverse image of every cozero set of Y is open in X.

DEFINITION 1.2.23 [11]

Let (X, T) and (Y, S) be any two topological spaces. A function $f : X \to Y$ is said to be Z-precontinuous at a point $x \in X$ if for every cozero set V of Y containing f(x), there exists a preopen set U in X such that $x \in U$ and $f(U) \subseteq V$.

The function is said to be Z-precontinuous if it is Z-precontinuous at each $x \in X$.

DEFINITION 1.2.24 [1]

A subset A of a topological space X is said to be pre Z-open if for each $x \in A$, there exists a copre-zero set H such that $x \in H \subset A$ or equivalently, A may be taken as the union of copre-zero sets.

The complement of a pre Z-open set is pre Z-closed and every copre-zero set is pre Z-open.

DEFINITION 1.2.25 [5]

Let (X, T) be a topological space and let $A \subset X$. A point $x \in X$ is said to be a Z-adherent point of A if every co-zero set containing x intersects A. The set of all Z-adherent points of A is denoted by AZ.

DEFINITION 1.2.26 [1]

Let (X, T) be a topological space and let $A \subset X$. A point $x \in X$ is said to be a pre Z-adherent point of A if every copre-zero set

containing x intersects A. Let APZ denote the set of all pre Z-adherent points of A. Since every co-zero set is a copre-zero set, $APZ \subset AZ$. A subset A of a topological space X is pre Z-closed if and only if $A = APZ$.

DEFINITION 1.2.27 [4]

Let (X, T) and (Y, S) be any two topological spaces. If f : X → Y is a function from X to Y, and A is a subset of X, then the restriction of f to A is the function f│A : A → Y having the graph G(f│A) = { (x, y) ∈ G(f)/x ∈ A }

DEFINITION 1.2.28 [8]

A subfamily T* of X is said to be supra topology on X if,

a) X, ϕ ∈ T*
b) If Ai ∈ T* for all i ∈ J, then ∪Ai ∈ T*.

(X, T*) is called a supra topological space. The elements of T* are called supra open sets in (X, T*) and complement of a supra open set is called a supra closed set.

DEFINITION 1.2.29 [8]

The supra closure of a set A is denoted by supra Cl(A) and defined as supra Cl(A) = ∩ { B: B is a supra closed and A ⊆ B }.

The supra interior of a set A is denoted by supra Int(A), and defined as supra Int(A) = ∪ { B: B is a T supra open and A ⊇ B }.

DEFINITION 1.2.30 [8]

Let (X, T) be a topological space and T* be a supra topology on X. we call T* a supra topology associated with T if T ⊂ T*.

DEFINITION 1.2.31 [8]

Let (X, T) and (Y, S) be two topological spaces. Let T* and S* be associated supra topologies with T and S respectively. Let f : (X, T*) → (Y, S*) be a map from X into Y, then f is a s-continuous function if the inverse image of each open set in Y is supra open in (X, T*).

DEFINITION 1.2.32 [3]

Let (X, T*) be a supra topological space. A set A is called supra semi-open set if A ⊆ supra Cl(supra Int(A)).

2.ON SUPRA Z-PRECONTINUOUS FUNCTIONS DEFINITION 2.1

Let (X, T*) be a supra topological space. A subset A of a space X is called a supra zero set if there exists a supra continuous functions f : (X, T*) → R, where R is defined to be a neighborhood of a real number x, such that A = {x ∈ X : f(x) = 0}. The complement of a supra zero set of a space X is called a supra co-zero set C of X.

DEFINITION 2.2

A subset A of a supra topological space X is called supra Z-open if for each x ∈ A, there exists a supra co-zero set C of X such that x ∈ C ⊂ A, or equivalently, if A can be expressed as the union of supra co-zero sets of X. The complement of a supra Z-open set is called supra Z-closed.

DEFINITION 2.3

A subset A of a supra topological space (X, T*) is called supra pre-open if A ⊆ supra Int (supra Cl(A)). Clearly every supra open set in X as well as a dense set in X is supra pre-open. The complement of a supra pre-open set of (X, T*) is called a supra pre-closed set. The family of all supra pre-open (supra pre-closed) sets of (X, T*) is denoted by SPO(X) (resp.SPF(X)).

EXAMPLE 2.1

Let X = { 1, 2, 3 }, P(X) = { X, ϕ, { 1 }, { 2 }, { 3 }, { 1, 2 }, { 2, 3 }, { 1, 3 } } is a power set of X.
Let T* = { X, ϕ, { 3 }, { 2, 3 } }.

Clearly, T* is a supra topology on X. The ordered pair (X, T*) is a supra topological space. Now, the supra closed sets of T* are ϕ, X, { 1, 2 }, { 1 }. Now, (X, T*) is a supra topological space. Let A = { 1, 3 } be a subset of (X, T*).

Then supra Cl(A) = X,
supra Int(supra Cl(A)) = X.
Hence, A ⊆ supra Int (supra Cl(A)).
Thus, A = { 1, 3 } is supra pre-open.

DEFINITION 2.4

Let (X, T*) be a supra topological space. For any set A in X, the intersection of supra pre-closed sets containing the set A is called the supra pre-closure of A and we denote it by SPCl(A) and SPCl(A) is a supra pre-closed set.
i.e., SPCl(A) = { B : B ⊇ A, B-supra pre-closed in X } is called supra pre-closure of A.

DEFINITION 2.4.1 [9]

Let (X, T*) be a supra topological space. For any set A in X, the union of supra pre-open sets containing the set A is called the supra pre-interior of A and we denote it by SPInt(A) and SPInt(A) is a supra pre-open set.

i.e., SPInt(A) = ⋃ { B : A ⊇ B, B-supra pre-open in X } iscalled supra pre-interior of A.

EXAMPLE 2.2

Let X = { a, b, c, d } and P(X)= { X, ϕ, { a }, { b }, { c }, { d },
{ a, b }, { b, c }, { c, d }, { b, d }, { a, d }, { a, b, c }, { a, b, d }, { b, c, d }, { c, d, a } } is a power set of X.
Let T* = { X, ϕ, { a }, { c }, { a, c }, { d }, { a, c, d }, { c, d }, { a, d } }.

Clearly, T* is a supra topology on X. The ordered pair (X, T*) is a supra topological space. Now, the supra closed sets of T* are ϕ, X, { b, c, d }, { a, b,d }, { b, d }, { a, b, c }, { b }, { a, b }, { b, c }.
Let A = { a } be a subset of X. Then, supra Int(A) = { a } supra Cl(supra Int(A)) = { a, b }.
Hence, A ⊉ supra Cl(supra Int(A)). Therefore, A = { a } is not supra pre-closed.
Let A = { c } be a subset of X. Then, supra Int(A) = { c }
supra Cl(supra Int(A)) = { b, c }. Hence, A ⊉ supra Cl(supra Int(A)).
Therefore, A = { c } is not supra pre-closed.
Let A = { d } be a subset of X.
Then, supra Int(A) = { d }
supra Cl(supra Int(A)) = { b, d }. Hence, A ⊉ supra Cl(supra Int(A)).
Therefore, A = { d } is not supra pre-closed.
Let A = { c, d } be a subset of X. Then, supra Int(A) = { c, d }
supra Cl(supra Int(A)) = { b, c, d }. Hence, A ⊉ supra Cl(supra Int(A)).

Therefore, A = { c, d } is not supra pre-closed.
Let A = { a, d } be a subset of X. Then, supra Int(A) = { a, d }
supra Cl(supra Int(A)) = { a, b, d }. Hence, A $\not\supseteq$ supra Cl(supra Int(A)).
Therefore, A = { a, d } is not supra pre-closed.
Let A = { a, c, d } be a subset of X. Then, supra Int(A) = { a, c, d }
supra Cl(supra Int(A)) = X.
Hence, A $\not\supseteq$ supra Cl(supra Int(A)).
Therefore, A = { a, c, d } is not supra pre-closed.

Since, every supra closed set is supra pre-closed set, the collection of all supra pre-closed sets is { X, ϕ, { b }, { b, d }, { a, b }, { b, c }, { a, b, c }, { a, b, d }, { b, c, d } }.
Let A = { c, d } be a subset of X.
Then, SPCl(A) = { b, c, d }.

Now, the supra pre-open sets of X are ϕ, X, { a, c, d }, { a, c }, { c, d }, { a, d }, { d }, { c }, { a }.
Let A = { b, d } be a subset of X. Then, SPInt(A) = { d }.

DEFINITION 2.5

A subset U of X containing a point x ∈ X is called a supra pre-neighborhood (or spre-nbd) of x in X if there exists A ∈ SPO(X) such that x ∈ A ⊂ U.

REMARK 2.1

Every supra pre-neighborhood A of x in X is supra pre-open in X.

DEFINITION 2.6

Let (X, T*) and (Y, S*) be any two supra topological spaces. A function f : (X, T*) → (Y, S*) is said to be supra pre-continuous at a point x ∈ X if for every supra open set V of Y containing f(x) there exists a supra pre-open set U in X, such that x ∈ U and f(U) ⊆ V. The function is said to be supra pre-continuous if it is supra pre-continuous at each x ∈ X.

DEFINITION 2.7

A subset A of a supra topological space X is said to be supra prezero set of X if there exists a supra precontinuous function f: (X, T*) → R such that A = { x ∈ X : f(x) = 0 } and is denoted by SPZ(f).

Its complement is called a supra coprezero set of X. Every supra zero set is a supra prezero set.

PROPOSITION 2.1

Every supra zero set is a supra prezero set.

PROOF

Let A be a supra zero set of X. Then there exist a supra continuous function f : (X, T*) → R such that A = { x ∈ X : f(x) = 0 }.

Since every supra continuous function is supra precontinuous, f : (X, T*) → R is a supra precontinuous function. Thus, there exist a supra precontinuous function f : (X, T*) → R such that

A = { x ∈ X : f(x) = 0 }.

Hence, Every supra zero set is supra prezero set.

DEFINITION 2.8

Let (X, T*) and (Y, S*) be any two supra topological spaces. A function f : (X, T*) → (Y, S*) is said to be supra Z-continuous if and only if the inverse image of every supra cozero set of Y is supra open in X.

DEFINITION 2.9

Let (X, T*) and (Y, S*) be any two supra topological spaces. A function f : (X, T*) → (Y, S*) is said to be supra Z-precontinuous at a point x ∈ X if for every supra cozero set V of Y containing f(x), there exists a supra preopen set U in X such that x ∈ U and f(U) ⊆ V.

The function is said to be supra Z-precontinuous if it is supra Z-precontinuous at each x ∈ X.

PROPOSITION 2.2

Let (X, T*) and (Y, S*) be any two supra topological spaces. For a function f : (X, T*) → (Y, S*), the following statements are equivalent:

a) f is supra Z-precontinuous.
b) The inverse image of every supra cozero set of Y is supra pre-open in X.
c) The inverse image of every supra zero set of Y is supra pre-closed in X.

PROOF

(a)⇒ (b) For any x ∈ X, f(x) ∈ Y. Let G be any supra cozero set in Y, containing f(x). Then by (a), for each x ∈ $f^{-1}(G)$, there exists a supra pre-open set U in X such that x ∈ U and f(U) ⊆ G. By definition, of supra pre-neighborhood, x ∈ f(U) ⊆ G which implies that, U ⊆ $f^{-1}(G)$.

Thus $f^{-1}(G)$ is a supra pre-neighborhood of each of its points. Therefore, f –1(G) is supra pre-open in X.

(b) ⇒ (c) Let A be any supra zero set in Y. Then Y – A is a supra cozero set in Y. Hence X–$f^{-1}(A)$ = $f^{-1}(Y)$ – $f^{-1}(A)$ and by (b), $f^{-1}(Y–A)$ = X – $f^{-1}(A)$ is supra pre-open in X. Therefore, $f^{-1}(A)$ is supra pre-closed in X.

(c) ⇒ (a) Let x ∈ X and Let G be any supra cozero set in Y containing f(x). Then f(x) ∉ Y – G and Y – G is a supra zero set. Since, $f^{-1}(Y – G)$ = $f^{-1}(Y)$ – $f^{-1}(G)$ and by (c), $f^{-1}(Y – G)$ = X – $f^{-1}(G)$ is a supra pre-closed set. Since, f(x) ∉ Y – G, x ∉ f –1(Y – G) and so x ∉ X – f –1(G). Thus f –1(G) is a supra pre-open set containing x and f($f^{-1}(G)$) ⊆ G. Therefore, f is supra Z-precontinuous.

DEFINITION 2.10

A supra metric on a set X is a function d : X × X → R having the following properties:

1) d(x, y) ≥0 for all x, y ∈ X; equality holds if and only if x = y.
2) d(x, y) = d(y, x) for all x, y ∈ X.
3) 3)(Triangle inequality) d(x, y) + d(y, z) ≥ d(x, z) for all x,y, z ∈ X.

DEFINITION 2.11

Let X be a supra topological space, X is said to be supra metrizable if there exists a supra metric d on the set X that induces the supra topology of X. A supra metric space is a supra metrizable space X together with a specific supra metric d that gives the supra topology of X.

REMARK 2.2

R is a supra metric space.

PROPOSITION 2.3

Let (X, T*) and (Y, S*) be any two supra topological spaces. For a function $f : (X, T^*) \to (Y, S^*)$, the following statements are equivalent:

a) f is supra Z-precontinuous.

b) The inverse image of every supra zero set of Y is supra prezero in X.

c) The inverse image of every supra cozero set of Y is supra coprezero in X.

PROOF

(a) ⇒ (b) For any $x \in X$, $f(x) \in Y$. Let f be supra Z-precontinuous and Z be any supra zero set in Y. Then, there exists a supra continuous function $g : (Y, S^*) \to R$ such that $Z = g^{-1}\{0\}$. Let F be any supra closed set in R.

Since in a supra metric space every supra closed set is a supra zero set, F is a supra zero set. As g is supra continuous $g^{-1}(F)$ is a supra zero set in Y. Since f is supra Z-precontinuous, $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is supra pre-closed in X. Hence $g \circ f$ is supra precontinuous. Now, since $g \circ f$ is supra precontinuous, $f^{-1}(Z) = f^{-1}(g^{-1}\{0\}) = (g \circ f)^{-1}\{0\}$ is a supra prezero set in X. Hence the inverse image of every supra zero set of Y is supra prezero in X.

(b) ⇒ (c) Let U be a supra cozero set in Y. Thus $Y - U$ is a supra zero set in Y. This implies that, f −1(Y − U) is a supra prezero set in X. Consider, $X - f^{-1}(U) = f^{-1}(Y) - f^{-1}(U) = f^{-1}(Y - U)$ is a supra prezero set, which implies that, $f^{-1}(U)$ is a supra coprezero set in X.

(c) ⇒ (a) Let U be a supra co-zero set in Y, which implies that, $f^{-1}(U)$ is a supra cozero set in X. This implies that, $f^{-1}(U)$ is a supra pre-open set in X. Therefore, f is supra Z-precontinuous.

DEFINITION 2.12

A subset A of a supra topological space X is said to be supra pre Z-open if for each $x \in A$, there exists a supra copre-zero set H such that $x \in H \subset A$ or equivalently, A may be taken as the union of supra copre-zero sets.

The complement of a supra pre Z-open set is supra pre Z-closed and every supra copre-zero set is supra pre Z-open.

DEFINITION 2.13

Let (X, T*) be a supra topological space and let $A \subset X$. A point $x \in X$ is said to be a supra Z-adherent point of A if every supra co-zero set containing x intersects A. The set of all supra Z-adherent points of A is denoted by ASZ.

DEFINITION 2.14

Let (X, T*) be a supra topological space and let $A \subset X$. A point $x \in X$ is said to be a supra pre Z-adherent point of A if every supra copre-zero set containing x intersects A. Let ASPZ denote the set of all supra pre Z-adherent points of A. By Proposition 2.1, every supra co-zero set is a supra copre-zero set. Hence ASPZ ⊂ ASZ. A subset A of a supra topological space X is supra pre z-closed if and only if A = ASPZ.

PROPOSITION 2.4

Let (X, T*) and (Y, S*) be any two supra topological spaces. For a function $f : (X, S^*) \to (Y, S^*)$, the following statements are equivalent:

a) f is supra Z-precontinuous.

b) The inverse image of every supra Z-open set of Y is supra pre Z-open in X.

c) The inverse image of every supra Z-closed set of Y is supra pre Z-closed in X.

PROOF

Similarly, the proof is as same as Proposition 2.3

PROPOSITION 2.5

Let (X, T*) and (Y, S*) be any two supra topological spaces. A function f : (X, T*) → (Y, S*) is supra Z-precontinuous if and only if for each A ⊂ X, f(ASPZ) ⊂ (f(A))SZ.

PROOF

Let f be supra Z-precontinuous. Then, the inverse image of every supra Z-closed set of Y is supra pre Z-closed in X. Let A ⊂ X, which implies that, (f(A))SZ is supra Z-closed in Y. Thus f^{-1}((f(A))SZ) is supra pre Z-closed in X. Now, f(A) ⊂ (f(A))SZ implies that, A ⊂ f^{-1}(f(A)) ⊂ f^{-1}(f(A))SZ and hence, ASPZ ⊂ f^{-1}(f(A))SPZ ⊂ (f –1((f(A)SZ)))SPZ = f^{-1}((f(A))SZ. Therefore, f(ASZ) ⊂ (f(A))SZ .Conversely, assume that f(ASPZ) ⊂ (f(A))SZ. Let E be a supra Z-closed set in Y. Then, f(f^{-1}(E)SPZ) ⊂ (f(f^{-1}(E)))SZ ⊂ ESZ = E, which implies that, f^{-1}(E)SPZ ⊂ f^{-1}(E). Thus, f^{-1}(E)SPZ = f^{-1}(E). This implies that, f^{-1}(E) is supra pre Z-closed in X. Therefore, f is supra Z-precontinuous.

PROPOSITION 2.6

Let (X, T*), (Y, S*) and (W, R*) be any three supra topological spaces. Let f : (X, T*) → (Y, S*) and g : (Y, S*) → (W, R*) be functions such that f is supra Z-precontinuous and g is supra continuous, then g ∘ f : (X, T*) → (W, R*) is supra Z-precontinuous.

PROOF

Let F be a supra zero set in W. Since g is a supra continuous, then g^{-1}(F) is a supra zero set in Y. Since f is a supra Z- precontinuous, f^{-1}(g^{-1}(F)) = $(g \circ f)^{-1}$(F) is a supra pre-closed set in X. Thus g ∘ f is supra Z-precontinuous.

DEFINITION 2.15

Let (X, T*) and (Y, S*) be any two supra topological spaces. If f : (X, T*) → (Y, S*) is a function from X to Y, and A is a subset of X, then the restriction of f to A is the function f│A : A → (Y, S*) having the graph G(f│A) = { (x, y) ∈ G(f)/x ∈ A }.

PROPOSITION 2.7

Let (X, T*) and (Y, S*) be any two supra topological spaces. If A ⊂ X, and f : (X, T*) → (Y, S*) is supra Z-precontinuous, then f│A : A → (Y, S*) is supra Z-precontinuous.

PROOF

Let V be any supra cozero set in Y. Then f –1(V) is a supra pre-open set in X. Thus (f│A)–1(V) = (f –1(V)) ∩ A is a supra pre-open set in A. Hence, f│A is supra Z-precontinuous.

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